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Huygens' principle in the transmission line matrix method (TLM). Local theory

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SUMMARY

Huygens' principle (HP) is a well-known fundamental principle of wave propagation. More generally, it can be understood as representing the principle of action-by-proximity (cf. Faraday's field theory etc.) and the superposition of secondary wavelets re-irradiated at each point of the wavefront (Huygens' construction). These wavelets are isotropic in free space and in isotropic materials. We will show, that HP is realized within the transmission line matrix method (TLM) for scalar fields in free space of any dimension, if one considers only the scattered fields to represent the secondary wavelets. This corrects and generalizes the previous result for the total field in 2D. This property of TLM provides another explanation for its wide range of applicability. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: TLM; scattering; Huygens' principle

1. INTRODUCTION

The transmission line matrix method (TLM) computes the propagation of short voltage or current pulses on a network of lossless transmission lines and lumped resistors [1–3]. It represents a wide range of transport and propagation problems, which are governed by the same laws. One of them is Huygens' principle (HP) in the sense of action-by-proximity (cf. Faraday's field theory etc.) and of the superposition of secondary wavelets (Huygens' construction [4]); for a precise formulation see Reference [5]. Each point of space reached by the wave front is considered to be excited and re-irradiating a secondary wave(let). The symmetry of this scattered wave is intimately related to the symmetry of the surrounding medium [6]. In particular, the scattered wave is isotropic in an isotropic medium.

The relationship of TLM to HP has been addressed as early as 1974 [8] (for a review, see Reference [7]). The secondary wavelets have been treated in terms of the intensity of the reflected and transmitted voltage (or current) pulses on the TLM network. However, isotropy is obtained only in case of a 2D lossless network, but not for a 3D lossless network. For, in the 3D case, the

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reflected voltage pulse amounts $(-2/3)V^{\mathrm{i}}$, while the transmitted ones equal $(1/3)V^{\mathrm{i}}$ in all directions, but that of the incident pulse, V^{i}. In this letter, we will show, that the isotropy of scattering is restored by discriminating between total and scattered fields, i.e. when the reflected pulse is treated as the superposition of incident and scattered fields.

2. HUYGENS' PRINCIPLE ON LOSSLESS TLM NETWORKS

Consider a node of a homogeneous 2D Cartesian lossless TLM network sketched in Figure 1 with an incident voltage pulse only from *West*, $V_{\mathrm{W}}^{\mathrm{i}}$ (as we will consider only scattering and not propagation, we suppress the time step index, k, as well as the node index). This pulse represents the incident field amplitude. The impedance discontinuity at the node gives rise to a reflected voltage pulse, $V_{\mathrm{W}}^{\mathrm{r}} = \rho_{\mathrm{TLM}}^{(2\mathrm{D})} \cdot V_{\mathrm{W}}^{\mathrm{i}}$, where the reflection coefficient is [7]

$$\rho_{\mathrm{TLM}}^{(2\mathrm{D})} = \frac{\frac{1}{3} - 1}{\frac{1}{3} + 1} = -\frac{1}{2} \tag{1}$$

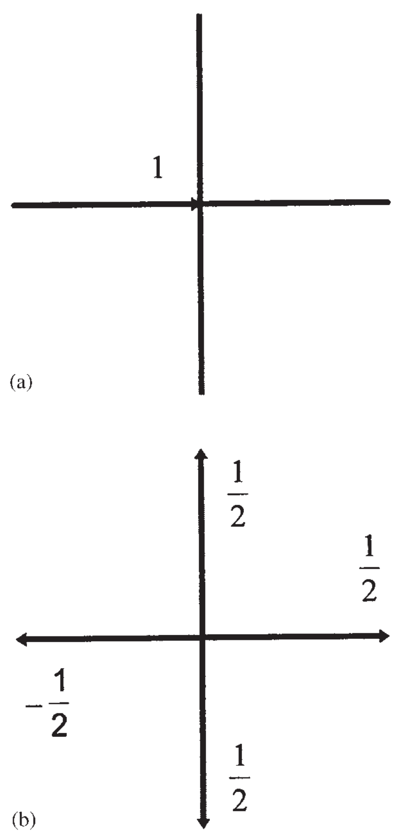

Figure 1. Common representation of scattering at the node of a lossless 2D cartesian TLM network [1, 3, 7]: (a) incident voltage pulse and (b) scattered voltage pulses. In all four directions, the intensity represented by the squared outgoing travelling voltage pulses equals the same value, 1/4.

Table I. Notions of incident, reflected, scattered and total fields.

Field	Previous notion	This letter
Incident field: total field before scattering	Incident pulse, V_A^{i}	Incident pulse, V_A^{i}
Reflected field: total field on the line of the incident field after scattering	Reflected, ${}_kV_A^{\mathrm{r}}$, (back-)scattered pulse, ${}_kV_A^{\mathrm{s}}$	Reflected pulse, $V_A^{\mathrm{r}}, V_A^{\mathrm{r}} = V_A^{\mathrm{bs}} + (-1)\cdot V_A^{\mathrm{i}*}$
Back-scattered field: scattered field on the line of the incident field	Reflected, ${}_kV_A^{\mathrm{r}}$ (erronous)	Back-scattered pulse, V_A^{bs}
Total field on the other line(s) after scattering	Transmitted/scattered pulse(s), $V_{B\neq A}^{\mathrm{s}}$	(Forward and sideward) transmitted pulse(s), $V_{B\neq A}^{\mathrm{t}}$

*The factor of '-1' accounts for the opposite direction of motion of incident and reflected fields.

The voltage pulses into the other three directions are $V_{\mathrm{N}}^{\mathrm{t}} = V_{\mathrm{E}}^{\mathrm{t}} = V_{\mathrm{S}}^{\mathrm{t}} = \tau_{\mathrm{line}}^{(2\mathrm{D})} V_{\mathrm{W}}^{\mathrm{i}}$, where

$$\tau_{\mathrm{TLM}}^{(2\mathrm{D})} = \frac{1}{3}(1 - \rho_{\mathrm{TLM}}^{(2\mathrm{D})}) = \frac{1}{2} \tag{2}$$

the *line* transmission coefficient, cf. Figure 1.

In previous treatments, isotropy of the reradiated wave(let) was stated as isotropy of intensity of the outgoing pulses as

$$\tau_{\mathrm{line}}^{(2\mathrm{D})^2} = \rho_{\mathrm{TLM}}^{(2\mathrm{D})^2} \tag{3}$$

It is easily seen, however, that this reasoning *fails* for a 3D node, where $\rho_{\mathrm{TLM}}^{(3\mathrm{D})} = -\frac{2}{3}$ and $\tau_{\mathrm{line}}^{(3\mathrm{D})} = \frac{1}{3}$, hence,

$$\tau_{\mathrm{line}}^{(3\mathrm{D})^2} \neq \rho_{\mathrm{TLM}}^{(3\mathrm{D})^2} \tag{4}$$

Why? The crucial point consists in the fact, that within this interpretation, the reflected pulses are identified with the (back-)scattered pulses. However, as matter of fact, the reflected pulses correspond to the *total* field amplitudes, i.e. to the sum of the incident and of the backscattered pulses. HP, however, refers to the scattered field only. We will now show, how HP is realized by the scattered voltage pulses. The missing differentiation between scattered field and total field being the sum of incident and scattered field is summarized in Table I.

The reflected pulse is the sum of the incident and of the back-scattered pulse.

$$V_{\mathrm{W}}^{\mathrm{r}} = V_{\mathrm{W}}^{\mathrm{bs}} + (-1)\cdot V_{\mathrm{W}}^{\mathrm{i}} \tag{5}$$

The factor (-1) accounts for the fact, that the incident and the back-scattered fields are moving in opposite directions. Thus, the back-scattering coefficient, ρ_{bs}, being defined through $V_{\mathrm{W}}^{\mathrm{r}} = \rho_{\mathrm{bs}}^{(2\mathrm{D})}\cdot V_{\mathrm{W}}^{\mathrm{i}}$ is given by

$$\rho_{\mathrm{bs}}^{(2\mathrm{D})} = \rho_{\mathrm{TLM}}^{(2\mathrm{D})} + 1 = \frac{1}{2} \tag{6}$$

Hence,

$$\tau_{\mathrm{line}}^{(2\mathrm{D})} = \rho_{\mathrm{bs}}^{(2\mathrm{D})} \tag{7}$$

It is this latter equality that states isotropic scattering referring to the amplitudes rather than to the intensities!

It is easily shown, that this result applies to any isotropic scalar node in any spatial dimension (nD), where

$$\tau_{\text{line}}^{(n\text{D})} = \rho_{\text{bs}}^{(n\text{D})} = \frac{1}{n} \tag{8}$$

For non-Cartesian lattices, where a node connects m equal transmission lines (e.g., $m = 6$ in a hexagonal lattice), one has

$$\tau_{\text{line}}^{(m)} = \rho_{\text{bs}}^{(m)} = \frac{2}{m} \tag{9}$$

3. DISCUSSION AND CONCLUSIONS

We have shown that the application of Huygens' Principle (HP) requires a careful discrimination of total and scattered fields. The emitters of the secondary wavelets are excited by the total field. Huygens construction, however, applies to the scattered field amplitude rather than to the total field intensity. The scattered field amplitude is isotropic in isotropic materials, including free space. Thus, HP in the sense of propagation by action-by-proximity and superposition of secondary wavelets applies to all scalar TLM nodes connecting an arbitrary number of transmission lines of equal impedance. This result corrects and generalizes the previous one for the total field in 2D. This property of TLM provides another explanation for its wide range of applicability. Finally, it will be interesting to investigate how this picture is modified when, (i) transmission lines of different impedances and (ii) losses are introduced.

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